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Nonlinear Modeling of Physiological Systems with Multiple Inputs

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Abstract: **Effective modeling of nonlinear dynamic systems can be achieved by employing Laguerre expansions and feedforward artificial neural networks in the form of the Laguerre-Volterra network (LVN). In this paper an extension of the LVN methodology to multiple-input systems is presented. Results from simulated systems show that this method can yield accurate nonlinear models of multiple-input Volterra systems, even when considerable noise is present.**
Keywords- **Volterra models, nonlinear modeling, multiple-input systems.**

I. INTRODUCTION

Previous modeling studies of nonlinear Volterra systems demonstrated the practical advantages of using Laguerre expansions of the Volterra kernels in order to achieve model compactness and estimation accuracy [1]. The resulting Laguerre expansion technique can be combined with feedforward artificial neural networks utilizing polynomial activation functions in the form of the Laguerre-Volterra network (LVN). The latter receives as its input vector the outputs of a Laguerre filter-bank fed by the input signal of the system, whereby the Laguerre parameter is estimated from the data [2].

A different formulation of the LVN, suitable for modeling nonlinear multiple-input systems, whereby each input is fed to a different Laguerre filter-bank, is examined herein. The use of different Laguerre filter-banks characterized by distinct parameters allows the effective study of the system dynamics related to each input separately as well as the study of their nonlinear interactions, quantified in the form of cross-terms. An illustrative example with synthetic data is presented and demonstrates the effectiveness of the proposed approach.

II. METHODOLOGY

The Laguerre-Volterra network (LVN) for a multiple-input input nonlinear system is shown in Fig. 1. Each of the inputs $\{x_i\}$ is fed into a different Laguerre filter-bank. The asymptotically exponential structure of the Laguerre functions makes them suitable for modeling physiological systems, since the latter often exhibit asymptotically exponential structure in their Volterra kernels. The Laguerre parameter a defines the relaxation rate of the Laguerre functions and determines the convergence of the Laguerre expansion for a given kernel function. Larger a values result in longer spread of significant values (slow dynamics). The choice of the Laguerre parameters $\{a_i\}$ for the filter-banks

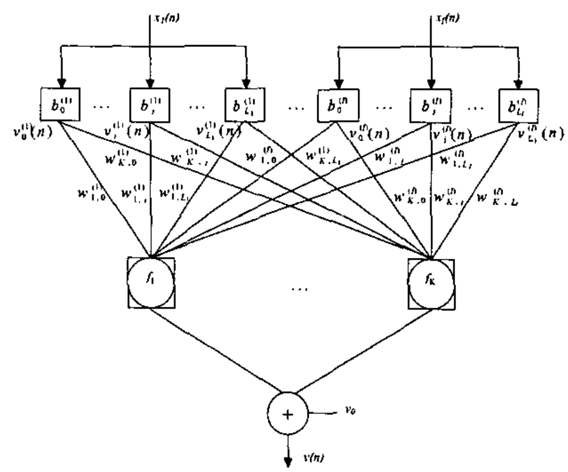

Figure 1: The multiple-input Laguerre-Volterra Network.

is very important in order to achieve an efficient model representation of the system under examination. We recently introduced a computationally efficient method, whereby the Laguerre parameters are treated as trainable parameters of the LVN [2].

The filter outputs in Fig. 1 are given by the discrete convolution of $b_j^{(i)}(m)$ with the corresponding input $x_i(n)$. The hidden units in the second layer have Qth order polynomial activation functions in order to make the network functionally equivalent to a Volterra model [3].

The training of all the network parameters is performed using a gradient descent iterative scheme, defining the cost function as the squared error between the desired output and the network output at each time instance n.

The difficulty in training the Laguerre parameters $\{a_i\}$ is tackled by employing the recursive relations for the computation of the Laguerre filter-bank outputs, given below for $j>0$:

$$v_j^{(i)}(n) = \beta_i[v_j^{(i)}(n-1) + v_{j-1}^{(i)}(n)] - v_{j-1}^{(i)}(n-1)\,;\, i = 1,\dots, I \quad (1)$$

where $\beta_i = a_i^{1/2}$, with the following initial condition:

$$v_0^{(i)}(n) = \beta_i v_0^{(i)}(n-1) + (1-\beta_i)^{1/2} x_i(n) \quad (2)$$

The total number of free parameters in the multiple-input LVN is equal to $(\sum_{i=1}^{I} L_i + I + Q)\cdot K + I + 1$. Note that this number is linear with respect to the order Q of the system, in contrast to other techniques (e.g., cross-correlation), the complexity of which depends on the system order Q exponentially. A Minimum Description Length

(MDL) criterion is employed to determine the values of the structural parameters in ascending order of successive trials.

The Volterra model for a Qth order nonlinear time-invariant discrete-time system with I inputs is:

$$y(n)=\sum_{n=0}^{Q}\sum_{i_1=1}^{\max(i_2)}\cdots\sum_{i_n=1}^{I}\left\{\sum_{m_1}\cdots\sum_{m_n}k_{i_1\ldots i_n}(m_1,\ldots,m_n)x_{i_1}(n-m_1)\ldots x_{i_n}(n-m_n)\right\} \quad (3)$$

If $i_1=\ldots=i_n$, $k_{i_1\ldots i_n}(m_1,\ldots,m_n)$ denote the nth order self-kernels of the system, otherwise they denote the nth order cross-kernels, which describe nonlinear interactions between different inputs. The Volterra kernels can be expressed in terms of the network parameters as follows:

$$k_0=y_0 \quad (4)$$

$$k_i(m_1)=\sum_{k=1}^{K}c_{1,k}\sum_{j=0}^{L_i}w_{k,j}^{(i)}b_j^{(i)}(m_1) \quad (5)$$

...

$$k_{i_1\ldots i_n}(m_1,\ldots,m_n)=\sum_{k=1}^{K}c_{n,k}\sum_{j_1=0}^{L_{i_1}}\cdots\sum_{j_n=0}^{L_{i_n}}w_{k,j_1}^{(i_1)}\ldots w_{k,j_n}^{(i_n)}b_{j_1}^{(i_1)}(m_1)\ldots b_{j_n}^{(i_n)}(m_n) \quad (6)$$

III. Results

The performance of the proposed approach is demonstrated with a high-order nonlinear system with two inputs, shown in Fig. 2. The linear filters L_1 and L_2 are characterized by the following impulse response functions:

$$l_1(m)=\exp(-\frac{m}{3})\sin(\frac{\pi m}{5}) \quad (7)$$

$$l_2(m)=\exp(-\frac{m}{20})-\exp(-\frac{m}{10}) \quad (8)$$

and the static nonlinearity N is of fourth order:

$$z_1(n)=2v_1(n)-3v_2(n)+v_1^2(n)-\frac{1}{2}v_2^2(n)+v_1(n)v_2(n)+\frac{1}{3}v_1^3(n)-\frac{1}{4}v_2^3(n)+\frac{1}{2}v_1^4(n)+\frac{1}{4}v_2^4(n) \quad (9)$$

The system is simulated for two independent Gaussian White Noise (GWN) stimuli of unit variance and a length of 512 points each. The simulation is then repeated after adding independent noise to the output (SNR=10 dB). Following an ascending search procedure with the MDL criterion, it is found that an LVN model with 7 Laguerre functions in each filter bank followed by three hidden units with fourth-degree polynomial functions is sufficient to model the system. The Normalized Mean Square Error (NMSE) of the prediction (defined as the model error sum-of-squares over the true output sum-of-squares) is equal to 0.12% in the noise-free case and 7.52% in the noisy case (ideally it should be 10%). The estimated two first-order Volterra kernels k_1 and k_2 are shown in Fig. 3 for both cases along with their true counterparts, demonstrating the excellent performance of the LVN method (kernel NMSEs 0.25% and 0.04% for k_1 and k_2 respectively in the noise-free case and 0.7% and 0.32% in the noisy case). The estimated second-order self and cross-kernels are also almost identical to their true counterparts in the noise-free case (NMSEs 0.31%, 0.9% and 0.54% for k_{11}, k_{22} and k_{12} respectively) and are affected more than the first-order kernels by the presence of noise (NMSEs 10.2%, 12.59% and 8.23% respectively). The Laguerre parameters of the two inputs converge to 0.283 and 0.743 in the noise-free case (0.289 and 0.715 in the noisy case), reflecting the respective fast and slow dynamics associated with the two inputs.

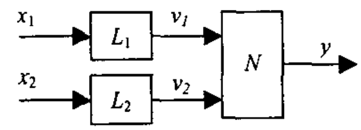

Figure 2: Simulated nonlinear system.

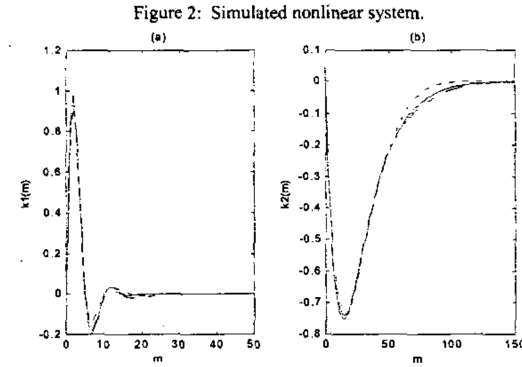

Figure 3: True and estimated first-order Volterra kernels: (a) $k_1(m)$ (b) $k_2(m)$. Solid: true, Dash-dot: noise-free output, Dash: noisy output (SNR=10 dB).

IV. Conclusion

The LVN approach can be used to model multiple-input nonlinear systems accurately using short input-output records, even in the presence of noise. An equivalent Volterra model can be obtained from the trained LVN that includes all nonlinear interactions among the various inputs. This methodology can find important application to multivariate physiological systems (e.g., the cerebral blood flow regulation system, to which the proposed approach was applied successfully).

References

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